

Comparison of different mathematical models for prediction of self-excited vibrations occurrence in milling process

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Abstract— In modern production, despite the existence of other production methods, metal cutting still plays an important role. The performance of machine tools has a decisive role in terms of productivity and quality of production increase. Undoubtedly, productivity and quality of production are two main requirements which are key elements to stay on top in a competitive market. One of the most influencing factor that affect the machine tools are vibrations. The most unwanted vibrations that can appear during metal cutting process are self-excited vibrations, which are one of the three kinds of mechanical vibration, free vibration, forced vibration, and self-excited vibration. When it comes to improving the performance of machine tools, the analysis of the appearance of self-excited vibrations and their isolation occupy a significant place. The aim of this paper derives from trends and limitations exists in metal production. The way to isolate the self-excited vibrations is to predict their occurrence by defining the stability lobe diagram. The paper presents two popular analytical methods for identifying stability lobe diagrams in milling, which shows the boundary between stable and unstable zone of machining operations, depending on the number of revolutions of the spindle and cutting depth. First considered method is Fourier series approach and second one is average tooth angle approach. Lather, both stability lobe diagrams were compared with results obtained experimentally. –

Key words –self-excited vibrations, stability lobe diagram, experimental modal analysis

I. INTRODUCTION

During a metal cutting processes, three types of vibrations can occur: free vibrations, forced vibrations and self-excited vibrations that affect the machining process to a greater or lesser extent. The most unfavorable vibrations that occur in the cutting process are self-excited vibrations, which energy for their formation and amplitude growth are drawn from the cutting process itself. These vibrations can occur due to friction in the system tool – workpiece, due to thermal or mechanical effects, or as a consequence of the regenerative effect, i.e. variation of the chip thickness during cutting. Self-excited vibration is a phenomenon that has a negative effect on productivity, leads to accelerated wear or breakage of the tool, and in some cases can lead to breakage of clamping elements or elements of the machine. In order to anticipate or control the process of creating self-induced vibrations, different methods have been developed. One way to predict the occurrence of self-induced vibrations is to create the stability lobe diagram (SLD), which defines the boundary of a stable, conditionally

stable and unstable region, whereby combinations of depth of cut, cutting speed i.e. number of revolution per minute and feed rate are observed.

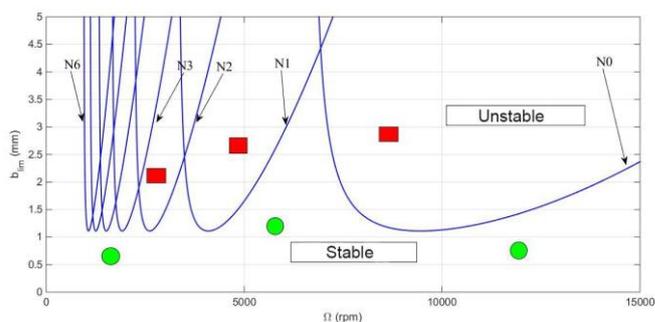


Figure 1. An example of a stability lobe diagram

Methods for defining the stability lobe diagram can be divided into analytical and experimental. Fig. 1, shows an example of a stability lobe diagram defined analytically. Two areas can be observed, "stable one", marked with a green marker and "unstable one" marked with a red marker.

A stable area, below the saddle curve lines, defines a combination of cutting parameters, that is, the number of revolutions and blim, the limiting chip width to avoid chatter, while the unstable region implies the occurrence of self-excited vibrations for the selected combination of the cutting parameters.

A. An analytical model for the prediction of self-induced vibrations

The analytical model for the prediction of self-excited vibrations requires the determination of transfer function of the mechanical structure of machine tool – cutting tool – workpiece. Namely, in order to define the stability lobe diagram by applying such models, some of the dynamic characteristics of the observed system needs to be known. In most cases, the necessary information on the dynamic characteristics of the system is obtained from the transfer function (TF) or from the Frequency Response Function (FRF) of system machine tool – tool holder – cutting tool. When determining the TF of main spindle - tool holder – cutting tool system, different methods can be applied. One of the methods is experimental modal analysis, in which the structure of the machine tool is excited by an impulse hammer, while response represents corresponding motion, speed or acceleration measured by appropriate sensors.

B. Experimental model of prediction of self-excited vibrations

The experimental model of prediction of self-excited vibrations can be observed as determination of the intervals of the certain parameters in which the cutting process is stable. Usually, workpiece has inclined plane, which causes the depth of cut gradually increase when the tool moves, until the moment of self-excited vibrations occurs. One of the methods for determining that self-excited vibrations occurs, shown in this paper, is to measure acceleration. Accelerometer is mounted on the main spindle carrier, as close as possible to the cutting tool. After self-excited vibrations occurred, which is manifested by the rapid jump of acceleration amplitude and by the change in sound of cutting, the process is stopped. Then, by the “tangent method” the axial depth of cut at which the vibrations have occurred has been determined. This paper analyses the prediction of self-excited vibrations in milling, using two popular analytical models, and presents the continuation of the research [1].

First analytical model, presented in [1], is based on assumption of “average” angle of the tooth in the cut, and second one, presented in this paper, is based on Fourier series approach. Also, an experimental verification of the analytically defined stability lobe diagram is carried out in a concrete example. The experimental part of this research is performed in the Laboratory for Machine Tools and CIM Systems at the Faculty of Mechanical Engineering in East Sarajevo on the vertical machining center EMCO Concept Mill 450. In order to determine modal parameters of the main spindle - tool holder – cutting tool system, which are necessary for the analytical definition of the stability lobe diagram, series of experiments are carried out using modern diagnostic equipment. Based on these experiments, frequency response function of the observed system is determined.

II. STATE OF THE ART

Research in the purpose of identification of self-excited vibration and their impact on stability of machine tools, were done by Tlustý [2] and Tobias [3] in the middle of the last century. They conducted their research in completely separate studies, but in almost the same time. Tlustý and Tobias proposed certain methods for analyzing the stability of machine tools such as determining the limiting chip width to avoid self-excited vibrations and stability lobe diagrams. When examining the stability of machine tools using limiting chip width, the great influence on the self-excited vibrations occurrence has cutting process parameters. Tlustý and Tobias have identified the regenerative mechanism of the self-excited vibrations and developed its mathematical model in the form of Delay Differential Equations (DDE). For defining the limiting chip width and creating a stability lobe diagram, it is necessary to know the transfer function, i.e. frequency response function of the main spindle - tool holder – cutting tool system. Tyler [4] describes an analytical solution for turning and milling stability including process damping effects and compares new analytical solution, time-domain simulation, and experiment. The method proposed by Altintas and Budak [5], the Zeroth Order Approximation (ZOA), is based on the prediction of system stability using Fourier's zero-order expression in order to approximate the change in the cutting force and the creation of a stability lobe diagram for process where the cutting force varies relatively little, e.g. in the machining of flat surfaces by face mill. Caixu [6] discussed the relationship between cutting stability and process damping, tool runout, and gyroscopic effect. Also, Caixu analysed how chatter in milling processes can be detected by using neural networks, Song [7], using the analytical and experimental approach, considers the influence of the number of teeth and how the tool helix angle affects on the appearance of self-excited vibrations, milling the alloy of aluminum Al 7075. Zataraina [8] considers the influence of tool helix angle on milling the aluminum alloy Al 7075 using end mills with different helix angles. Quintana [9] defines a stability lobe diagram in milling experimentally, whereby milling inclined surface workpiece is performed. The occurrence of self-excited vibrations is registered by recording and analyzing sound emissions, using modern signal processing techniques. Based on FFT sound analysis, the moment when self-excited vibrations appear is determined, which excludes the possibility of an error due to the subjective feeling of the operator. In addition, a large number of researchers also applied complex mathematical expressions to model self-excited vibrations in order to define the stability lobe diagram. Insperger and Stephan [10] applied a semi-discrete method (Semi-discretization, SD) to reduce the DDE method to a series ordinary differential equations (ODE) with a known solution. In his research, Gradišek [11] compared the boundaries of the stability of the milling process obtained by the methods of ZOA and SD, and concluded that these two methods give very similar stability lobe diagrams for machining with high radial immersion. On the other hand, for low radial immersion, significant differences can be noticed in defined stability lobe diagrams. Stepan [12] analyses stability of turning for flexible workpiece varying dynamic properties due to the material removal process by finite element method. The Frequency Response Function is determined depending on tool position. The results show significant change of modal parameter, i.e. natural frequency and modal stiffness during cutting, which have significant influence on vibration.

III. ANALYTICAL MODEL FOR DEFINITION OF THE STABILITY LOBE DIAGRAM BASED ON THE AVERAGE TOOTH ANGLE APPROACH

An analytical model for defining a stability lobe diagram in milling based on the average tooth angle approach, shown in this paper, is a modified Tlustý model for turning to accommodate the milling process [2]. The difference between these two analytical models is that on turning cutting force is time invariant, while on milling is variable and time-dependent. The analytic model of defining the stability lobe diagram in turning was modified to the milling by introducing the average tooth angle approach, which may result in the milling process as time invariant.

According to the analytical model of defining the stability lobe diagram [2], the relationship between the depth of cut which self-excited vibrations occur in milling and number of revolutions are given by the

$$b_{lim} = \frac{-1}{2 \cdot K_s \cdot \cos(\beta) \cdot \text{Re}[FRF_{orient}] \cdot N_t^*} \quad (1)$$

$$\frac{f_c}{\Omega \cdot N_t} = N + \frac{\varepsilon}{2 \cdot \pi} \quad (2)$$

In these equations

b_{lim} – is the limiting chip width to avoid chatter

f_c – is the chatter frequency

N – is the integer number of waves of vibration imprinted on the workpiece surface in one revolution

N_t – average number of teeth in the cut, given by the equation

$$N_t^* = \frac{\phi_e - \phi_s}{\frac{360}{N_t}} \quad (4)$$

In the previous equations ϕ_e and ϕ_s are exit and start angle, $\varepsilon/2\pi$ is any additional fraction of a wave, where is ε the phase (in rad) between current and previous tool vibrations.

The oriented FRF of machine tool main spindle – tool holder – cutting tool system is calculated by summing the products of the directional orientation factors and corresponding FRFs for the x and y directions;

$$FRF_{orient} = \mu_x \cdot FRF_x + \mu_y \cdot FRF_y \quad (5)$$

The directional orientation factors μ_x and μ_y are determined depending on the relative movement between the tool and the workpiece (up milling or down milling), and exit and start angle ϕ_e , ϕ_s in the next way: cutting force F is first projected into x and y axis, which determines the components of the force F_x and F_y . These components are then projected into surface normal passing through the axis of the tool, Fig. 2.

For the slotting cut where $\phi_s = 0$, $\phi_e = 180^\circ$, and radial immersion is 100% the average angle of a tooth in the cut is therefore given by the next equation

$$\phi_{sr} = \frac{\phi_s + \phi_e}{2} = \frac{0 + 180}{2} = 90^\circ \quad (6)$$

The directional orientation factors are

$$F_x = F \cdot \cos(\beta) \quad (7)$$

$$F_n = F_x \cdot \cos(0) = F \cdot \cos(\beta) \cdot \cos(0) = F \cdot \cos(\beta) \quad (8)$$

$$\mu_x = \cos(\beta),$$

$$F_y = F \cdot \cos(90 - \beta) = F \cdot \sin(\beta) \quad (9)$$

$$F_n = F_y \cdot \cos(90) = F \cdot \sin(\beta) \cdot \cos(90) = 0 \quad (10)$$

$$\mu_y = 0$$

The oriented FRF is calculated by

$$FRF_{orient} = \cos(\beta) \cdot FRF_x + 0 \cdot FRF_y \quad (11)$$

meaning that compliance in the y direction has no influence on the stability.

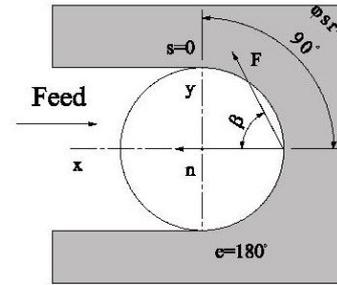


Figure 2. Determining slotting directional orientation factors

IV. ANALYTICAL MODEL FOR DEFINITION OF THE STABILITY LOBE DIAGRAM BASED ON THE FOURIER SERIES APPROACH

Altintas and Budak [5] use slightly different methods than previously explained to express equations describing the milling process as time invariant. In this method, the time varying coefficients of the dynamic milling equations are expanded into a Fourier series. The Fourier series approach was developed considering a tool to be a dynamic system with two degrees of freedom, which has N teeth and helix angle of 0, Fig. 3.

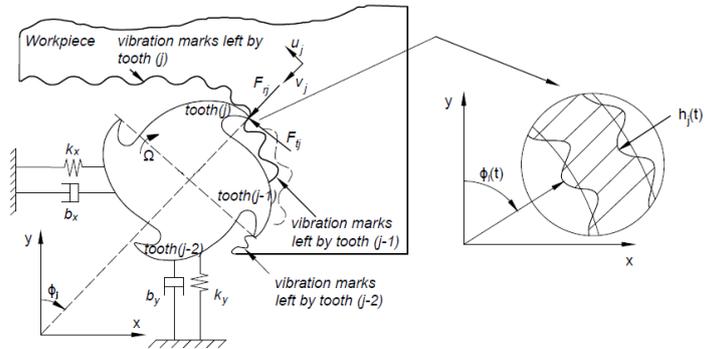


Figure 3. Dynamic model of milling [5]

According to this model, limiting chip width b_{lim} is determined by

$$b_{lim} = \frac{2\pi\Lambda_R}{NK_t} (1 + \kappa^2)$$

$$n = \frac{60}{NT}$$

where A_R and A_I are real and imaginary part of complex quadratic function defined by FRF, $\kappa = \frac{\Lambda_I}{\Lambda_R}$, T represents the period between two consecutive tool passes and is equal $T = (\varepsilon + 2k\pi) / \omega_c$, k represent number of waves on the stability lobe diagram, ω_c angular velocity of self-excited vibrations, $\varepsilon = \pi - 2\psi$ phase shift between waves on a workpiece formed by two consecutive tooth, where $\psi = \tan^{-1} \kappa$, Ks specific force value in tangent direction.

For the known cutting tool geometry, specific force value, transfer function of the system tool-tool holder-main spindle and frequency of self-excited vibrations ω_c , is possible to determine Λ_I and Λ_R , and thus the corresponding limiting chip width and the main spindle speed. A stability lobe diagram for the milling is obtained when this procedure is repeated for a certain range of frequencies of self-excited vibrations and the number of "waves".

In order to define the stability lobe diagram of any of the analytical methods mentioned above, it is necessary to know some of the dynamic characteristics of the observed system, namely its modal parameters. In most cases, the necessary information about the dynamic characteristics of the system is obtained from the transfer function (TF) or from the frequency response function (FRF) of the tool-tool holder-main spindle system.

The most commonly way in determining the TF of the tool – tool holder – main spindle system is impact force testing, whereby the structure, i.e. tool mounted into tool holder and main spindle is excited by an impulse hammer while the response (displacement, speed, or acceleration) is measured by appropriate sensor.

V. EXPERIMENTAL MODAL ANALYSIS

In order to define the stability lobe diagram of the EMCO Concept Mill 450 vertical machining center by described analytical method, it is necessary to determine the modal parameters of the next system: machine tool main spindle - tool holder – cutting tool. Modal parameter needs to be determined are natural frequency, modal stiffness and dimensionless damping ratio. The modal analysis represents the process of identification of dynamic characteristics, i.e. modal parameters. To identify modal parameters, a large number of procedures are applied, which can be roughly classified into methods based on the application of analytical methods, numerical methods, and experimental methods. The analytical way of identifying modal parameters implies accurate mathematical solutions that describe the behavior of a construct in dynamic operation, which is possible only in simple and idealized cases, which are mostly not met in practice. The numerical way of identifying modal parameters most often involves the application of finite elements method (FEM) software. Dynamic parameters, apart from analytical and numerical, can be determined by experimental testing. Experimental tests are a very desirable method for determining modal parameters since

they are, in general, simple, fast, and very important in the research, there is no material destruction. Experimental modal analysis involves simultaneously analyzing the signal of the excitation and the response in the frequency domain. This research shows the determination of the modal parameters of the system machine tool main spindle - tool holder – cutting tool based on the experimental modal analysis. Fig. 4 shows model for determining FRF of vertical machining center Emco Concept Mill 450.

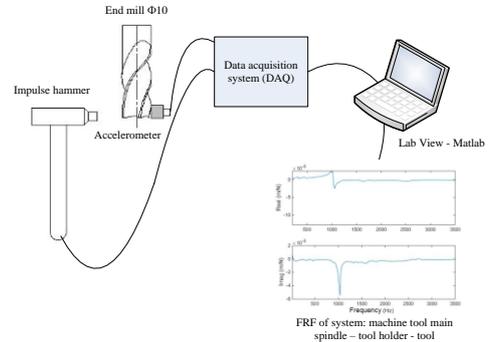


Figure 4. Model for determining FRF of vertical machining center

In the tool holder, an end mill $\Phi 10$ with four teeth is mounted by ER collet clamping system. Impact force is applied on the top of the tool, and response is measured by an accelerometer. Acquisition is carried out using LabView software. After completed experimental measurements, the collected data are transferred to the Matlab environment, where the FRF is defined by fast Fourier transform (FFT) and represents by its "real-imaginary" part. Fig. 5. Shows FRF of system machine tool mail spindle - tool holder – tool represented by its real and imaginary part, and in Table 1, the parameters necessary for defining the stability lobe diagram obtained by the experimental modal analysis are given.

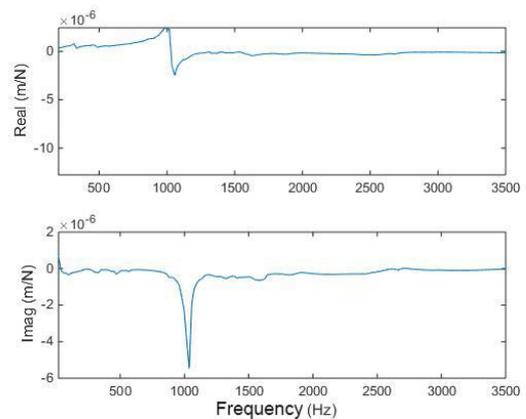


Figure 5. Real and imag part of FRF

TABLE I. PARAMETERS NEEDED FOR ANALYTIC DEFINING THE STABILITY LOBE DIAGRAM LE TYPE STYLES

| | |
|--|-------------------|
| $\varphi_s = 0$ | Start angle |
| $\varphi_e = 180$ | Exit angle |
| $f_{nx} = 1053\text{Hz}$ $f_{ny} = 1016\text{Hz}$ | Natural frequency |

| | |
|--|------------------------------|
| $kx = 1,5e8 N / m$ $ky = 5,3e7 N / m$ | Stiffness |
| $\zeta x = 0,026$ $\zeta y = 0,017$ | Dimensionless damping ratio |
| $d = 10 mm$ $N=4$ | Diameter and number of tooth |
| $K_s = 750 N/mm^2$ | Specific force |

VI. EXPERIMENTAL VERIFICATION OF ANALYTICALLY DEFINED STABILITY LOBE DIAGRAM

The experimental part of this research is carried out at the vertical machining center Emco Concept Mill 450 in the Laboratory for machine tools and CIM systems at the Faculty of Mechanical Engineering in East Sarajevo. In milling, a hard metal end mill with four teeth with a coating of Ti, Al (N) and two helix angles, 35° and 38° is used. The methodology for defining a stability lobe diagram implies the implementation of a series of experiments, whereby workpiece has the surface with a 10° slope, Fig. 6.

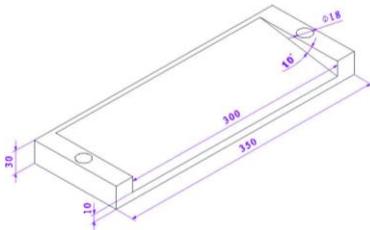


Figure 6. Shape and dimensions of the workpiece

In this way, when tool moves, the depth of cut gradually increases until the moment of self-excited vibration occurs. Due to the cutting depth change, a very slight increase in the amplitude of the cutting tool vibrations occurs, and at a time when the depth of cut reaches the limiting chip value, a sudden jump of amplitude occurs, indicating the self-excited vibrations presence. The vibration parameter directly measured by the contact method is acceleration, using thereby the National Instruments instrumentation.

The instrumentation consists of a National Instruments cDAQ 9172 chassis and an analog NI 9233 card with four analog inputs of a ± 5 V voltage range and a maximum signal speed of $50 \text{ kS} / \text{s}$ (kilograms per second). METRIX Instruments accelerometer, sensitivity $100 \text{ mV} / \text{g}$ with piezo-ceramics, is mounted on the main spindle carrier using a magnetic holder, as close as possible to the tool. When acquiring data, the graphical programming language LabVIEW is used, and for processing the results of measurement Matlab environment. It should also be noted that when calculating the measurement results in order to determine the limiting chip value, the absolute value of the vibration amplitude is not taken into account because it is not relevant for determining the moment when self-induced vibrations occur.

This moment, apart from the sudden vibration amplitude jump, is characterized by a change in the surface quality and chip shape, as well as the appearance of intensive sound, which is an indicator that the machine works with unfavorable cutting regime. At the appearance of sound, the operator stops the operation of the machine, but the axial depth of cut is

subsequently determined using the tangent method. On the basis of the obtained results of measurements in the time domain, the axial depth of cut is determined in which the self-excited vibrations occur, in the following way.

In the area of the amplitude jump, the polynomial is withdrawn passing through the vibration peaks. At the point where the polynomial has the highest value of the first derivative, the tangent is withdrawn. The cross-section of the tangent with the horizontal line that is pulled through the vibrations peaks in the part of a stable processing process (in which no self-excited vibrations) represents the moment when self-excited vibration occurs, and determines the axial depth of cut δ . Namely, based on a known time interval from the moment of contact of the cutting tool with the material, until the moment of the self-excited vibrations occurs (Fig. 5, value $t_2 - t_1$), and the known federate and the angle of inclination of the workpiece, axial depth of cut at which self-excited vibration occurs has been calculated. In Fig. 6. signal in the time domain was shown as an illustration of described method, as well as the moment of self-excited vibration appears for an experiment performed at 5750 rpm and feed rate $460 \text{ mm} / \text{min}$.

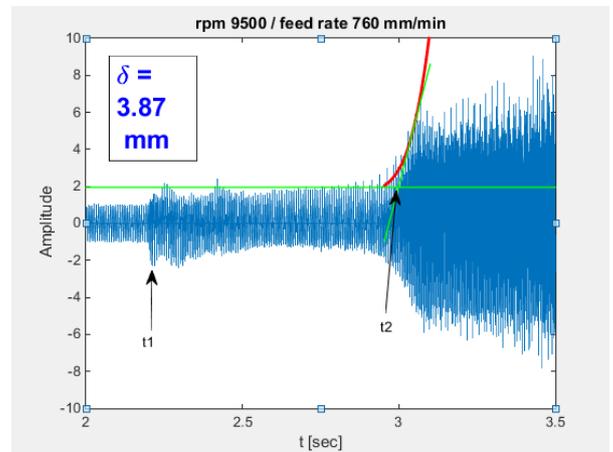


Figure 7. Time domain signal at 5750 rpm and feed rate $460 \text{ mm} / \text{min}$

From the Fig. 7 can be seen that, in the concrete case, the depth of cut at which the self-excited vibrations appear is $\delta = 3.30 \text{ mm}$. If self-excited vibrations did not appear, the experiment is interrupted at the moment of reaching the maximum defined depth of cut. Since the diameter of the tool is 10 mm and that the experiment was carried out in the slotting cut where, and radial immersion is 100% , in order to prevent the possible breakage of the tool, the experiment is interrupted when depth of cut reaches 8 mm , if self-excited vibration was not occurred before.

Fig. 8 shows the workpiece after the experimentally defined stability lobe diagram. Fig. 9 shows comparison of the stability lobe diagram defined analytically, with experimentally defined one. Experimentally determined axial depths of cut, shown in the Fig. 9 are marked with red markers. The defined stability lobe diagrams are shown in two dimensions (2D), because all experiments are performed for one, constant value of feed per tooth. Table 2 gives the values of the speed of cut (revolution per minute), feed rate, and axial depth of cut determined for all experiments carried out.



Figure 8. Workpiece after experimentally defined stability lobe diagram

The experimentally defined axial depth of cut, where zone of stable and unstable cut are clearly visible, were used to confirm the analytically defined stability lobe diagram for aluminum alloy Al 7075 milling on the vertical machining center, in the observed range of the revolutions per minute, and at the constant values of feed rate.

TABLE II. AXIAL DEPTH OF CUT DEPENDING OF REVOLUTION PER MINUTE WITH CONSTANT FEED PER TOOTH VALUE

| Number of revolutions (rpm) | Feed rate (mm/min) | Axial depth of cut (mm) | Vibration occurred |
|-----------------------------|--------------------|-------------------------|--------------------|
| 2000 | 160 | 7.95 | No |
| 2250 | 180 | 8.00 | No |
| 2500 | 200 | 8.00 | No |
| 2750 | 220 | 8.00 | No |
| 3000 | 240 | 8.00 | No |
| 3250 | 260 | 3.46 | Yes |
| 3500 | 280 | 6.10 | Yes |
| 3750 | 300 | 8.00 | No |
| 4000 | 320 | 8.00 | No |
| 4250 | 340 | 3.35 | Yes |
| 4500 | 360 | 7.22 | Yes |
| 4750 | 380 | 8.00 | No |
| 5000 | 400 | 8.00 | No |
| 5250 | 420 | 6.41 | Yes |
| 5500 | 440 | 3.86 | Yes |
| 5750 | 460 | 3.30 | Yes |
| 6000 | 480 | 5.09 | Yes |
| 6250 | 500 | 4.68 | Yes |
| 6500 | 520 | 8.00 | No |
| 6750 | 540 | 8.00 | No |
| 7000 | 560 | 8.00 | No |
| 7250 | 580 | 8.00 | Yes |
| 7500 | 600 | 8.00 | No |
| 7750 | 620 | 8.00 | No |
| 8000 | 640 | 6.61 | Yes |
| 8250 | 660 | 4.07 | Yes |
| 8500 | 680 | 4.17 | Yes |
| 8750 | 700 | 2.74 | Yes |
| 9000 | 720 | 4.13 | Yes |
| 9250 | 740 | 3.36 | Yes |
| 9500 | 760 | 3.87 | Yes |
| 9750 | 780 | 3.00 | Yes |
| 10000 | 800 | 5.59 | Yes |
| 10250 | 820 | 5.39 | Yes |
| 10500 | 840 | 4.70 | Yes |
| 10750 | 860 | 5.59 | Yes |
| 11000 | 880 | 6.71 | Yes |
| 11250 | 900 | 7.12 | Yes |
| 11500 | 920 | 5.19 | Yes |

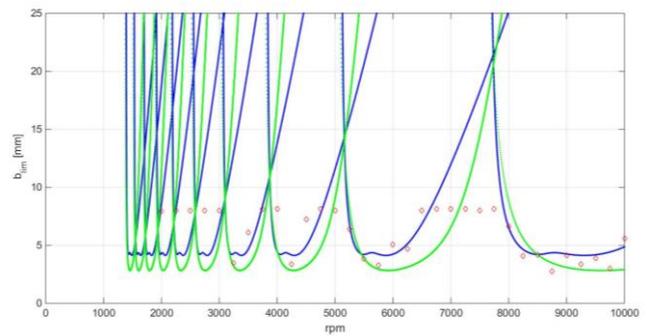


Figure 9. Stability lobe diagram defined analytically and experimentally (green line – Fourier series approach, blue line – average tooth angle approach, red markers – experimentally obtained data)

VII. CONCLUSION

Analyzing the results, i.e. comparing the analytically defined stability lobe diagrams with experimentally obtained one, can be concluded that the considered analytical models yields good results, and that the stability lobe diagram obtained in such a way, can be reliably used in the definition of milling operations.

A stability lobe diagram, as a boundary between a stable and unstable cutting, is a function of number of revolutions and axial depth of cut. This paper presents two analytical methods for defining the stability lobe diagram based on the average tooth angle approach and Fourier series approach and the experimental verification of the obtained results. The boundary between a stable and unstable cut can be determined by measuring the axial depth of cut on the workpiece in which self-excited vibrations have occurred, where the areas of stable and unstable cut are clearly visible. By processing the collected signals in the time domain and using the tangent method, axial depth of cut can be determined, in which the cutting process becomes unstable. The methodology presented is suitable for determining the stability lobe diagram for slot milling where radial immersion is 100% and average angle of tooth is 180°.

Fig. 9 shows stability lobe diagram obtained the average tooth angle (blue line), and diagram obtained by Fourier series approach (green line). On the same figure, limiting chip width b_{lim} to avoid chatter obtained experimentally are marked by red marker. Limiting chip width b_{lim} was determined based on conducted experiments, by the “tangent method”.

It can be seen that limiting depth of cut under given conditions obtained by the Fourier series approach has slightly lower values, by about 1 mm. It can be seen that in the lower speed range, up to about 3000 rpm, there is no major difference in the shape of the curves obtained by both analytical methods.

At spindle speed above 3000 rpm, the saddle curve line obtained by the Fourier series approach is a slightly wider width than the curve obtained by average tooth angle method. In the same time the curve obtained by the average tooth angle method is better aligned with the experimentally obtained results, but only for the spindle speed lower than 7000 rpm. For the spindle speed above 7000, Fourier series approach provides better fits experimentally obtained results than stability lobe obtained by average tooth approach. It is necessary to note that

this consideration is only worthy of the given conditions, i.e. 100% radial immersion (slotting).

It is also useful to look back on the time needed to create a stability lobe diagram. Both analytical models request several parameters that should be known, or determined. Primarily, specific force K_s and modal parameters like natural frequency, dimensionless damping ratio, as well as FRF (Frequency Response Function). Specific force K_s can be found in different literature, but modal parameter, from the other side, needs to be determined for each condition, i.e. each tool replacement. Having in mind that modal parameters could be determined only experimentally, it is obviously that this request some time, specific equipment an experience.

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tools, flexible technological

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