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Application of Particle Swarm Optimization for Classical Engineering Problems

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Abstract—**In the design of mechanical elements, designers usually consider certain objectives that are related with cost, time, quality and reliability of product, depending on the requirements. In this paper, parametric optimization of spring design problem, pressure vessel design problem, cantilever beam design problem, cone coupling design problem and welded beam design problem has been carried out using Particle Swarm Optimization (PSO for short). The pseudo code for this algorithm was written using Matlab R2018a software suite. Results of the PSO algorithm will be compared to results obtained by the Differential Evolution (DE), Modified Ant Colony Algorithm, (MACA), Grasshopper Optimization Algorithm (GOA), Water Cycle Algorithm (WCA), Cucko Search (CS) , Genetic Algorithm (GA), Ant Lion Optimization (ALO), Firefly Algorithm (FA) and Method of Moving Asymptotes (MMA), depending of solutions found in literature. The source code of Particle Swarm Optimization (PSO) algorithm is publicly available at https://seyedalimirjalili.com.**

Keywords-optimization, pso algorithm, spring, pressure vessel, cantilever beam, cone coupling, welded beam

I. INTRODUCTION

Metaheuristics are an impressive area of research, with extremely important improvements, and are used for solving intractable optimization problems. Major advances have been made since the first metaheuristic was proposed and numerous new algorithms are still being proposed every day. There is no doubt that the studies in this field will continue to develop in the near future. In the field of metaheuristics, there is a set of algorithms which draw inspiration from nature, so called biologically-inspired metaheuristic algorithms. The main characteristic of this class of algorithms is using a population of search agents to explore the solution space, in order to find the best solution possible.

In this paper, we will apply PSO for solving classical problems in engineering. This algorithm was selected because of it having few parameters to adjust, robustness, high efficiency in finding the global optima, and fast convergence.

The first problem [1] consists of minimization of spring weight subject to constrains on minimum deflection, shear stress, surge frequency, limits on the outside diameter and design variables. The design variables are: coil diameter $D(x_1)$, wire diameter $d(x_2)$ and number of active coils $N(x_3)$. This problem was solved by Differential Evolution (DE) algorithm in [13], Genetic Evolution (GE) algorithm in [12], and Modified Ant Colony Optimization (MOCO) in [15].

The second problem is optimization of a pressure vessel, which consists of reducing costs of material, montage and welding costs. Four variables are defined for this problem: radius of shell(x_1), length of the shell(x_2), thickness of the shell(x_3) and thickness of the dish end(x_4) [2]. This problem was solved by Differential Evolution (DE) algorithm in [13], Modified Ant Colony Optimization (MACO) in [15], and Grasshopper Algorithm [14].

The third engineering problem is cantilever beam optmization, where minimal weight that fulfills the constraints is sought after. Gandomi has solved this problem using the Cuckoo Search Algorithm (CSA)[3]. The results of PSO algorithm are compared with those obtained by ALO [16], MMA [17] and GOA [18].

The fourth engineering problem that will be considered in this paper is optimization of a cone coupling. The goal of this optimization is to minimize coupling volume. This example was defined in [4]. The results of PSO algorithm are compared with those obtained by FA [19], CS [19] and H-CS-FA [19].

The last problem is welded beam optimization in which it is necessary to optimize the minimum cost subject to constraints on shear stresses, bending stresses in the beam, buckling load, end deflection of the beam and other side

This paper is a revised and expanded version of the paper presented at the XIX International Symposium INFOTEH-JAHORINA 2021, [31] contraints described by Rao [5]. This problem was solved by hybrid Genetic Algorithm [20], Genetic Algorithm [12], and Water Cycle Algorithm [21].

II. PARTICLE SWARM OPTIMIZATION

PSO algorithm was developed in 1995. by Eberhart and Kennedy [6]. It took little time for this algorithm to attract attention of many researchers, and is still used for solving engineering problems.

In paper by Manickavelu and Vaidyanathan [7], the PSO algorithm was used to make predictions about route rediscovery during route failures in mobile networks. The network consisted of nodes, whose status was decided upon by fuzzified parameters. This method was tested on a randomized network, while the packet size and node speed were varied. The PSO has shown better results in all the test cases. In paper by Nanni and Lumini [8], the PSO was used for improving performance of ensembling generation for evidentional KNN classifier. This was done using a random subspace based ensembling method. Given a set of random subspace evidential kNN classifier, a PSO is used for obtaining the best parameters of the set of evidential k-nearest-neighbour classifiers, finally these classifiers are combined by the vote rule. This method was tested on several benchmark datasets, showing better performance by having smaller error rate. In paper by Meissner et al. [9], an improvised version of PSO algorithm, called Optimized PSO, was demonstrated. Instead of having one swarm, the swarm is divided into subswarms, which perform PSO. The subswarms are used for solving the optimization problem, while the superswarm is responsible for optimizing subswarm parameters. OPSO has shown better results in terms of speed and robustness in comparison to the standard PSO. In paper by Lu et al. [10], the ILSPSO algorithm was demonstrated, and a self-learning strategy is used to improve the standard PSO. This method was tested on several benchmark functions, showing better results than all other tested PSO variants. In paper by Zhang et al. [11], a multi-objective problem was solved by the modified PSO algorithm, called Niche PSO. The problem involved mapping virtual networks to substrate networks, in terms of revenue and energy cost. The Niche PSO has shown better results for both objective functions, while having a slightly larger execution time.

The phenomenon from which this algorithm draws inspiration is very interested. It is based on simulating the motion of a group of particles moving in solution space, where the position of a particle represents a solution of the problem. Since the algorithm is dealing with a group of solutions, it belongs to the class of metaheuristic algorithms that are called population-based (or p-based) metaheuristics. The whole of the particles is called population. By moving the particles their variables values change, and tracking, controlling, and directing these particles help them reach the optimum.

Characteristic variables that are necessary for the realization
of this algorithm are position and speed. A particle's position in a given moment represents a potential solution, while only the current best position is memorized and leads the optimization process.

New solution is based on Equation (1).

$$X_{New,i} = X_{Old,i} + \upsilon_{New,i} \quad (1)$$

Having :

$$\upsilon_{New,i} = \omega \cdot \upsilon_{Old,i} + C_p \cdot r_p \left(X_{p,i} - X_{x,i} \right) + C_g \cdot r_g \left(X_{g,i} - X_{x,i} \right) \quad (2)$$

In Equation (2), ω represents particle inertia, while C_p and C_g represent acceleration factors. Acceleration factors are positive-valued constants which control the local influence for the given particle and the global direction for the given particle. Variables r_p i r_g are assigned random values between 0 and 1, and are used to vary search along the whole problem space. The variable X_{pi} represents the best position of a given particle, variable X_{gi} represents the best position for the whole population, while the variable X_{xi} represents the current position.

Value ω is calculated by using Equation (3).

$$\omega = \omega_{max} - \frac{\omega_{max} - \omega_{min}}{Iteration_{max}} \cdot Iteration \quad (3)$$

In Equation (3), ω_{max} and ω_{min} represent the initial and final values for inertia. The recommended values for these parameters are 0.9 and 0.4, respectively. The values for ω must therefore be in the range between 0 and 1. The values for C_p i C_g are adjusted according to researchers' experience and the literature, and their recommended values are 1.5 for both constants. There is much research that focus on examining the algorithm's efficiency with regards to the coefficients, with the fore mentioned value of 1.5 being but one of the many recommended values.

In order to demonstrate the workings of PSO algorithm, the crux of the algorithm, that is the motion of particles, is given in Fig. 1.

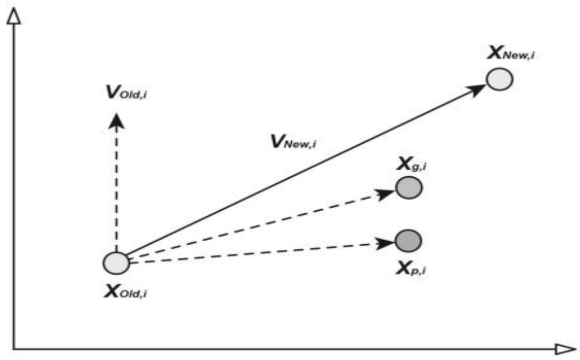

Figure 1. Graphic representation of particle movement[4]

This algorithm was applied to the practical optimization problems in engineering as well [22-24], most notably to the problems of structural optimization. The analysis of these problems was performed by using the standard PSO algorithm, as well as its modifications and hybrid algorithms.

Around 2010, the largest use for this algorithm was in the field of multicriterial optimization problems [25-28]. The Integrated Particle Swarm Optimization (IPSO for short) algorithm, and a hybrid of Genetic algorithm and Particle Swarm Optimization, called Genetic Algorithm Particle Swarm Optimization (GAPSO for short) were used for practical management engineering problems [29]. The literature mentioned in this paper represents only a small part of research literature focused on PSO algorithm.

Flow diagram for the PSO algorithm is shown in Fig. 2.

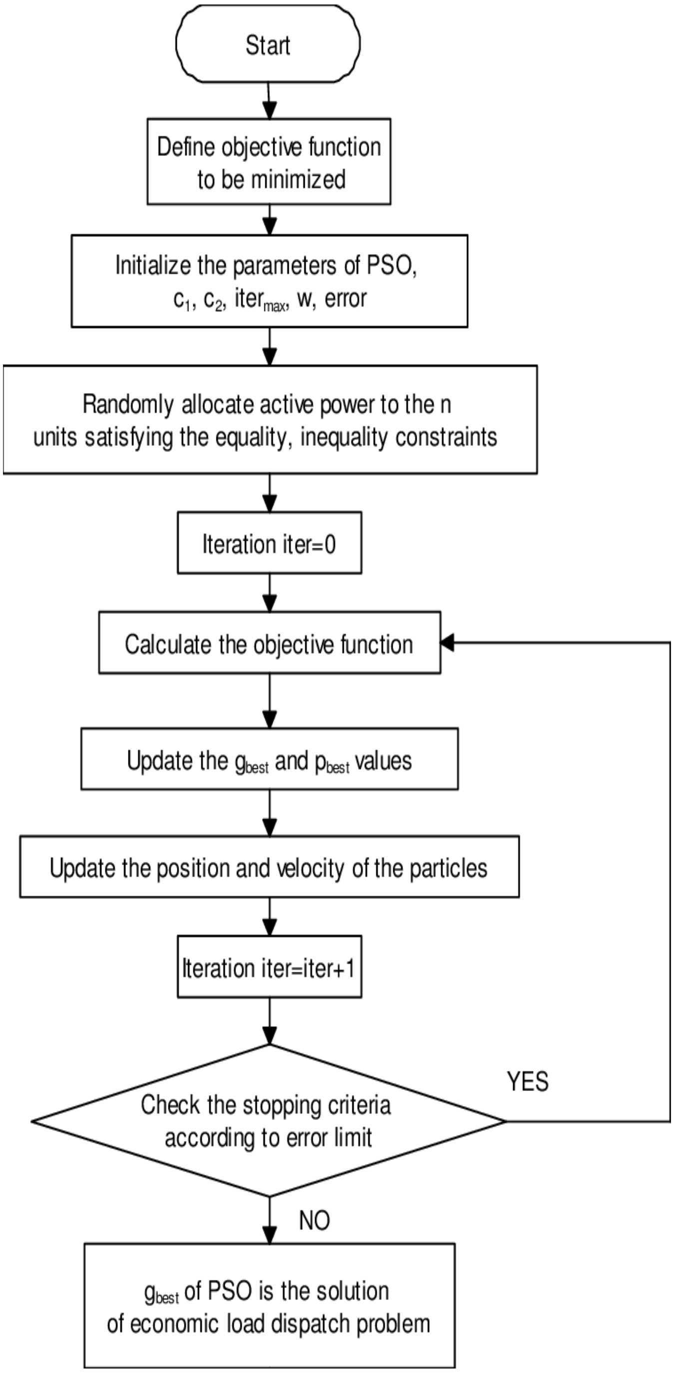

Figure 2. Flow diagram for PSO algorithm[4]

The pseudo code for the algorithm is given below:

I) **For** each particle:
Initialize particles.
II) **Do**:
a) **For** each particle:
1) Calculate fitness value
2) If the fitness value is better than the best Fitness value (pBest) in history
3) Set current value as the new pBest
End

b) **For** each particle:
1) Find in the particle neighborhood, the particle With the best fitness
2) Calculate particle velocity according to the Velocity equation
3) Apply the velocity constriction
4) Update particle position according to the Position equation
5) Apply the position constriction
End
While maximum iterations or minimum error criteria is not attained.

III. EXPERIMENTAL ENGINEERING EXAMPLES FOR OPTIMIZATION

The main problem with these five examples is to find the minimum optimal solution which must satisfy a series of given constraints.

The optimization problem having only one objective function can be formulated in the following manner:

$$\begin{aligned} &\min/\max f(x), \\ &g_j(x) \le 0, \quad j = 1,2,\ldots,J; \\ &h_k(x) = 0, \quad k = 1,2,\ldots,K; \\ &x_i^G \ge x_i \ge x_i^D, \quad i = 1,2,\ldots,N. \end{aligned} \quad (4)$$

Where:

$f(x)$ - objective function

$x = [x_1\, x_2 \ldots x_N]^T$ - vector of problem variables

$g_j(x)$ - inequality-type constraints

$h_k(x)$ - equality-type constraints

x_i^D - lower bound for x_i

x_i^G - upper bound for x_i

This chapter will present certain examples of engineering problems, such as: optimization of helical spring, pressure vessel, cantilever beam, cone coupling and welded beam. The basis of the problem, the objective function, variable parameters that should be found as well as the constraints that should be respected will be shown.

The optimum design of helical spring problem is to minimize the volume of the spring (Fig. 3) under four non-linear constraints.

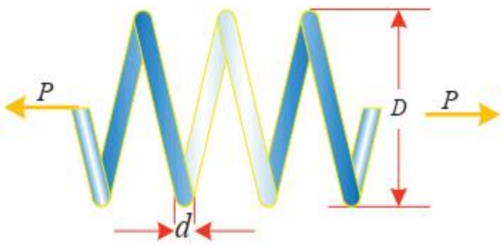

Figure 3. Helical spring design[13]

Formally, the first problem can be expressed as minimization of the function $f(x)=(x_3+2)x_2x_1^2$, defined in [1], subject to the following constraints:

$$g_1(x)=1-\frac{x_2^3x_3}{71785x_1^4}\leq 0; \quad (5)$$

$$g_2(x)=\frac{4x_2^2-x_1x_2}{12566\left(x_2x_1^3-x_1^4\right)}+\frac{1}{5108x_1^2}-1\leq 0; \quad (6)$$

$$g_3(x)=1-\frac{140,45x_1}{x_2^2x_3}\leq 0; \quad (7)$$

$$g_4(x)=\frac{x_1+x_2}{1,5}\leq 0; \quad (8)$$

$$0,05\leq x_1\leq 2; \quad (9)$$

$$0,25\leq x_2\leq 1,3; \quad (10)$$

$$2\leq x_3\leq 15; \quad (11)$$

The pressure vessel problem (Fig. 4) must be designed for minimum total fabrication cost subject to four constraints.

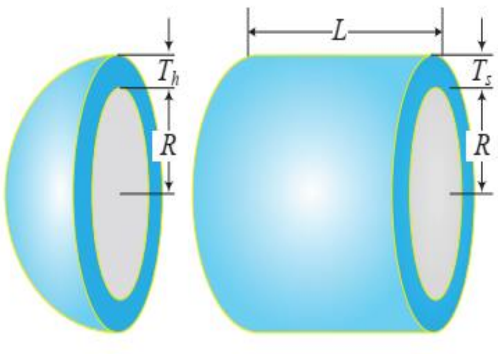

Figure 4. Cylindrical pressure vessel design[13]

Objective function to be minimized, as defined in [2]:

$$f(x)=0,6224x_1x_3x_4+1,7781x_2x_3^2+3,1661x_1^2x_4+19,84x_1^2x_3 \quad (12)$$

$$g_1(x)=-x_1+0,0193x_3\leq 0; \quad (13)$$

$$g_2(x)=-x_2+0,00954x_3\leq 0; \quad (14)$$

$$g_3(x)=-\pi x_3^2x_4-\frac{4}{3}\pi x_3^3+1296000\leq 0; \quad (15)$$

$$g_4(x)=x_4-240\leq 0; \quad (16)$$

In Fig. 5 a schematic view of speed cantilever beam is shown.

As seen in Fig. 5, the cantilever beam consists of five hollow, box shaped bearings with a square shaped frame. Project variables are lengths of the five squares(x_1,x_2,x_3.x_4 x_5,) which make up the cantilever beam.

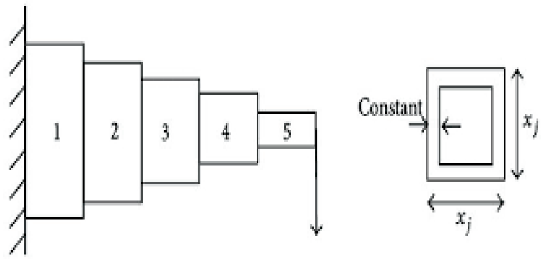

Figure 5. Cantilever beam design problem[21]

Goal function to be minimized is defined as:

$$f(\vec{x})=0,6224(x_1+x_2+x_3+x_4+x_5), \quad (17)$$

Whilst the only constraint for this problem being:

$$g(\vec{x})=\frac{61}{x_1^3}+\frac{27}{x_2^3}+\frac{19}{x_3^3}+\frac{7}{x_4^3}+\frac{1}{x_5^3}-1\leq 0, \quad (18)$$

$$0,01\leq x_1,x_2,x_3,x_4,x_5\leq 100, \quad (19)$$

The cone coupling problem (Fig. 6) must be designed for minimum volume coupling to two constraints. Problem variables are: inner radius of the coupling $R_1\equiv x_1$ and outer radius of the coupling $R_2\equiv x_2$

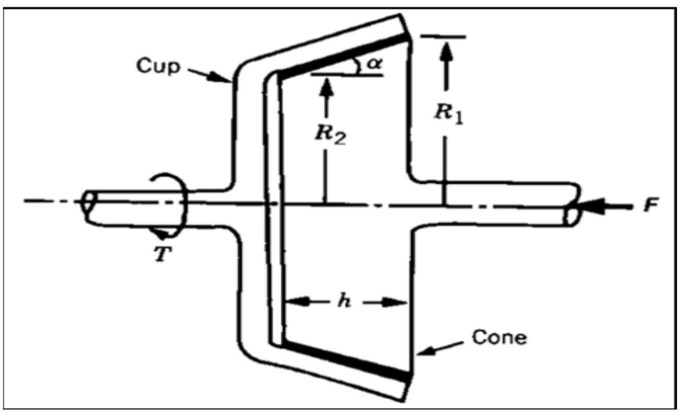

Figure 6. Cone coupling design problem[19]

Goal function to be minimized is defined as:

$$f(X) = \left(x_1^3 - x_2^3\right) \quad (20)$$

Whilst the conditions to be met are:

$$g_1(X) = \frac{x_1}{x_2} \geq 2 \quad (21)$$

$$g_2(X) = \frac{\left(x_1^2 + x_1 x_2 + x_2^2\right)}{\left(x_1 + x_2\right)} \geq 5 \quad (22)$$

$$1 \leq x_1, x_2 \leq 10 \quad (23)$$

The welded beam problem (Fig. 7) must be designed for minimum manufacturing cost subject to seven constraints. The four variables that should be optimized which are: the size of the weld h(x1), the length of the welded section of the beam l(x2), the beam width t(x3) and the beam thickness b(x4).

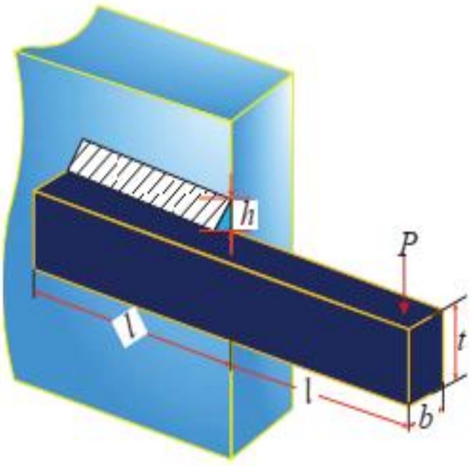

Figure 7. Welded beam design problem[13]

The problem consists of minimization of the function

$$f(x) = 1{,}10471 x_1^2 x_2 + 0{,}04811 x_3 x_4 \left(14 + x_2\right)$$

subject to the following constraints:

$$g_1(x) = \tau(x) - \tau_{max} \leq 0 \quad (24)$$

$$g_2(x) = \sigma(x) - \sigma_{max} \leq 0 \quad (25)$$

$$g_3(x) = x_1 - x_4 \leq 0 \quad (26)$$

$$g_4(x) = 0{,}10471 x_1^2 + 0{,}04811 x_3 x_4 \left(14 + x_2\right) - 5 \leq 0 \quad (27)$$

$$g_5(x) = 0{,}125 - x_1 \leq 0 \quad (28)$$

$$g_6(x) = \delta(x) - \delta_{max} \leq 0 \quad (29)$$

$$g_7(x) = P - P_c(x) \leq 0 \quad (30)$$

$$0{,}125 \leq x_1 \leq 10\ ;\ 0{,}1 \leq x_2 \leq 10$$
$$0{,}1 \leq x_3 \leq 10\ ;\ 0{,}1 \leq x_4 \leq 5 \quad (31)$$

where:

$$\tau(x) = \sqrt{\left(\tau'\right)^2 + 2\tau'\tau''\frac{x_2}{2R} + \left(\tau''\right)^2} \quad (32)$$

$$\tau' = \frac{P}{\sqrt{2} x_1 x_2} \quad (33)$$

$$\tau'' = \frac{MR}{J} \quad (34)$$

$$M = P\left(L + \frac{x_2}{2}\right) \quad (35)$$

$$R = \sqrt{\frac{x_2^2}{2} + \left(\frac{x_1 + x_3}{2}\right)^2} \quad (36)$$

$$J = 2\left\{\sqrt{2} x_1 x_2 \left[\frac{x_2^2}{12} + \left(\frac{x_1 + x_3}{2}\right)^2\right]\right\} \quad (37)$$

$$\sigma(x) = \frac{6PL}{x_4 x_3^2} \quad (38)$$

$$\delta(x) = \frac{4PL^3}{E x_3^3 x_4} \quad (39)$$

$$P_c(x) = \frac{4{,}013E\sqrt{\frac{x_3^2 x_4^6}{36}}}{L^2}\left(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right) \quad (40)$$

$P = 6000 lb$; $L = 14 in$; $E = 30 \times 10^6\ psi$;
$G = 12 \times 10^6\ psi$; $\tau_{max} = 13600\ psi$;
$\sigma_{max} = 30000\ psi$; $\delta_{max} = 0{,}25 in$.

IV. RESULTS AND DISCUSSION

In this section, the results obtained by using PSO algorithm on previously defined engineering problems is given.

Based on results shown in Table 1, a conclusion can be drawn that the objective function having the value of 0.01268, that is obtained using the PSO algorithm, is close to other values found in literature.

TABLE I. COMPARISON OF RESULTS BETWEEN PSO AND OTHER ALGORITHMS FOR HELICAL SPRING

Objective function	Abderazek [13]	Coello [12]	Grkovic [15]	**PSO**
f(x)	0.01266	0.01268	0.01265	**0.01268**

In Fig. 8, a convergence diagram for the problem of helical spring optimization is given.

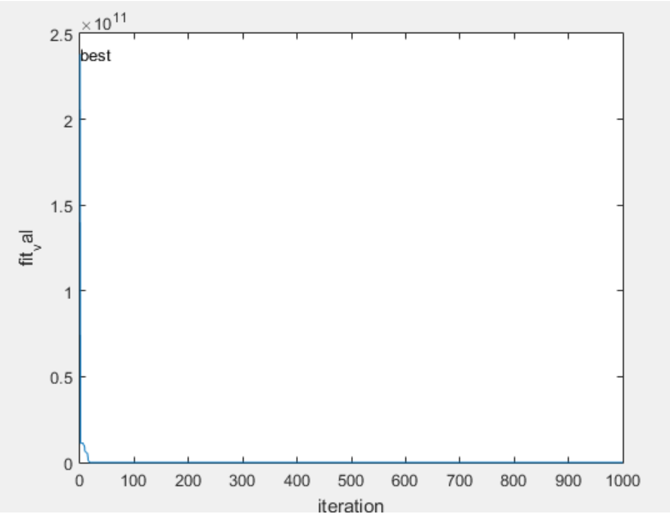

Figure 8. Convergence graph for the best solution for helical spring design

In Table 2, a comparison of results for design of a pressure vessel optimization problem are shown. In [13], Differential Evolution Algorithm is used, while paper [12] uses Genetic Algorithm, and paper [14] uses Grasshopper Optimization Algorithm.

TABLE II. COMPARISON OF RESULTS BETWEEN PSO AND OTHER ALGORITHMS FOR PRESSURE VESSEL

Objective function	Abderazek [13]	Coello [12]	Jovanovic [14]	**PSO**
f(x)	6059.714	6288.74	7665.12	**5885.33**

PSO algorithm achieved better result than Abderazek, Coello and Jovanovic.

In Fig. 9, a convergence diagram for the problem of pressure vessel optimization is given.

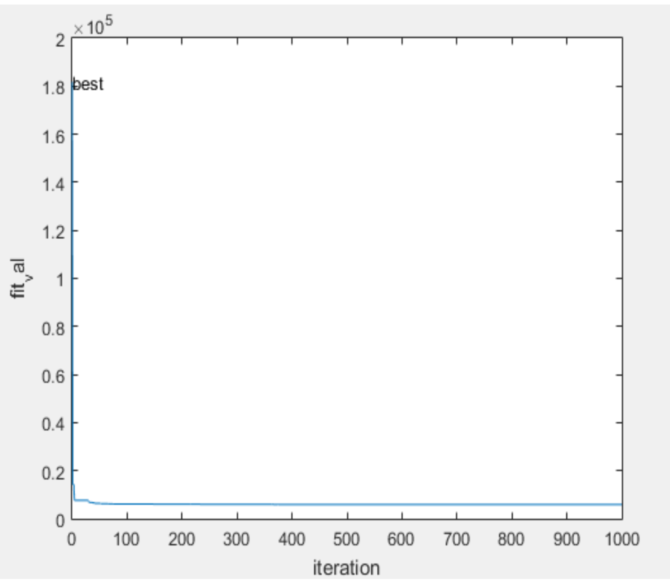

Figure 9. Convergence graph for the best solution for pressure vessel design

For the cantilever beam design problem, the results shown in Table 3, along with the results obtained by ALO, MMA and GOA methods.

TABLE III. COMPARISON OF RESULTS BETWEEN PSO AND OTHER ALGORITHMS FOR CANTILEVER BEAM

Objective function	ALO [16]	MMA [17]	GOA [18]	**PSO**
f(x)	1.339	1.340	1.339	**1.339**

PSO gives better result in comparison to MMA, while in comparison to ALO and GOA the results are the same.

In Fig#. 10, a convergence diagram for the problem of cantilever beam optimization is given.

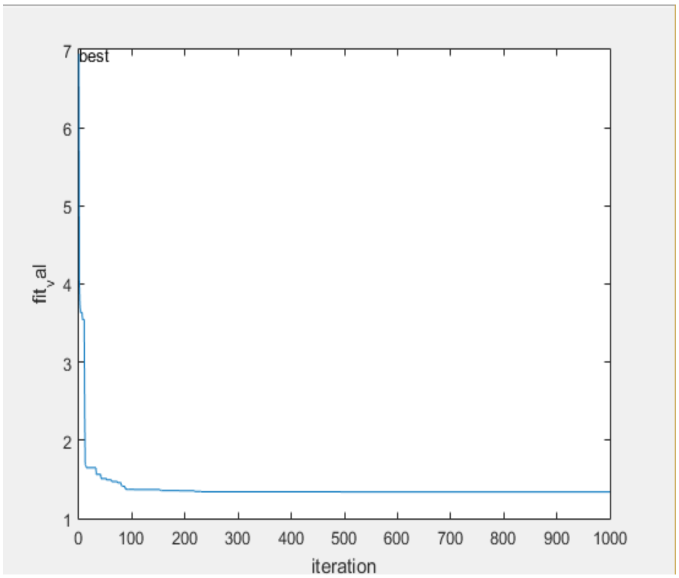

Figure 10. Convergence graph for the best solution for cantilever beam design

In Table 4, a comparison of results for design of a cone coupling optimization problem are shown. Analysing the table results, a conclusion has been drawn that the PSO gives better result in comparison to FA, while in comparison to CS and H-CS-FA the results are nearly the same.

TABLE IV. TABLE 4. COMPARISON OF RESULTS BETWEEN PSO AND OTHER ALGORITHMS FOR CONE COUPLING

Objective function	FA [19]	CS [19]	H-CS-FA [19]	**PSO**
f(x)	69.62784	68.88715	68.88215	**68.87755**

In Fig. 11, a convergence diagram for the problem of cone coupling optimization is given.

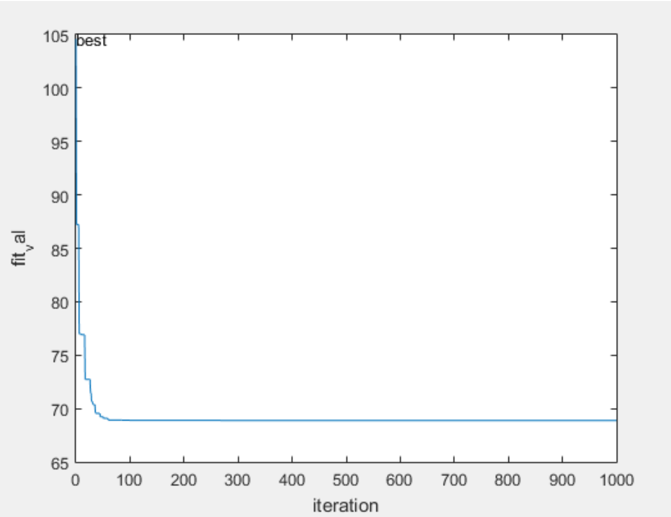

Figure 11. Convergence graph for the best solution for cone coupling design

A detailed presentation of the results obtained by the PSO method and comparison of several best results obtained by using other algorithms are given in Table 5.

TABLE V. TABLE 5. COMPARISON OF RESULTS BETWEEN PSO AND OTHER ALGORITHMS FOR WELDED BEAM

Objective function	Zhao [20]	Coello [12]	Eskandar [21]	**PSO**
f(x)	1.724852	1.748309	1.724856	**1.724852**

Analysing the table 5 results, a conclusion has been drawn that the PSO gives better result in comparison to Coello, while in comparison to Eskandar and Zhao the results are nearly the same.

In Fig. 12, a convergence diagram for the problem of welded beam optimization is given.

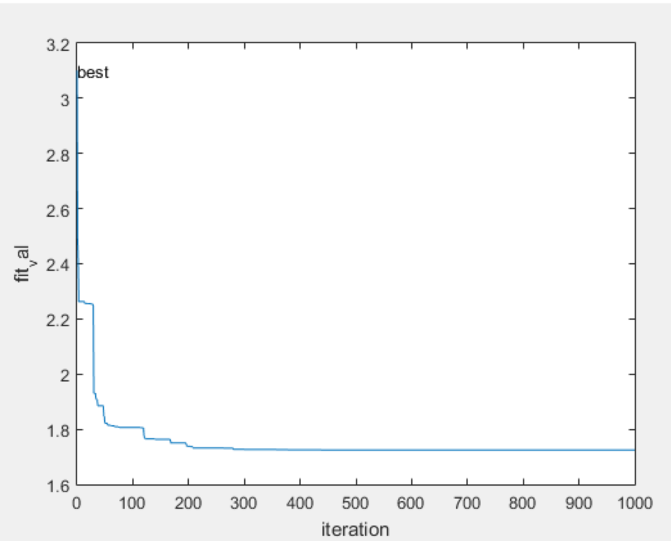

Figure 12. Convergence graph for the best solution for welded beam design

V. CONCLUSION

This paper describes the PSO algorithm, as well as its application in few engineering problems. The mentioned engineering problems of helical spring, pressure vessel, cantilever beam, cone coupling and welded beam are given in detail, using mathematical formulation and figures, and the results are given in tables.

For this algorithm, 300 search agents and 1000 iterations were chosen as input parameters. During the course of the research, it has been noted that increasing search agent and iteration count did not yield better solutions. Therefore, this combination of input parameters was chosen, since it gives minimal execution time.

In the case of pressure vessel optimization, the PSO algorithm gives better results than other methods found in literature. The results for the other four optimization problems, namely the cantilever beam, helical spring, cone coupling and welded beam problem, were shown to be near optimal.

Therefore, solving these problems using the new optimization technique presented in this paper provides an important opportunity for researchers to compare the performances of their new methods using complex mechanical engineering design optimization problems.

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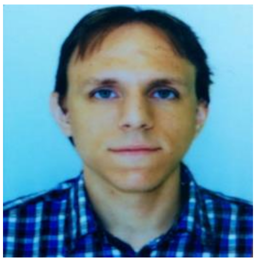

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